S&P 500 – Adaptive Rebalancing

(Part 3)

By

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S&P 500 vs. US Government Bonds

• During the period 1978-2013 the average annualized return was almost 6\% for US Government Bonds with 1 year maturity.
• The bond returns are guaranteed by the government of USA.
• The average annualized return for the S&P 500 was 11-13\% depending on investment duration.
• But the S&P 500 was very volatile with a standard deviation over 17\% for annual returns. The greatest annual gain was over 70\%, the greatest annual loss was almost (50\%).
Fixed vs. Adaptive Rebalancing

• Rebalance between S&P 500 and US Gov. Bonds to lower volatility.
• Each year the portfolio is rebalanced back to the desired allocation.
• Fixed rebalancing uses a predetermined allocation e.g. 50 / 50 or 25 / 75.
• It is simple but does not take the price-level of the S&P 500 into account.
• There is a relation between the P/Book (Price-To-Book ratio) and long-term returns of the S&P 500. (See another talk for more on this.)
• Adaptive rebalancing uses P/Book to adjust the portfolio allocation.
The stock-weight is the part of the portfolio invested in the S&P 500. It is calculated using the P/Book of the S&P 500. The formula is:

$$\text{Stock Weight} = \text{Limit}(1.5 - 0.5 \times \text{P/Book})$$

Limited between zero and one.

Rebalancing is only done annually.
Example: Calculate the Stock Weight

On January 12, 1999 the P/Book was 4.64 so the stock-weight was:

\[ \text{Stock Weight} = \text{Limit}(1.5 - 0.5 \times \text{P/Book}) = \text{Limit}(1.5 - 0.5 \times 4.64) = \text{Limit}(-0.82) = 0 \]

So the portfolio should be invested entirely in US Gov. Bonds.

The P/Book was high in several years. Then in 2003 it was 2.89:

\[ \text{Stock Weight} = \text{Limit}(1.5 - 0.5 \times \text{P/Book}) = \text{Limit}(1.5 - 0.5 \times 2.89) = \text{Limit}(0.06) \approx 0.06 \]

In 2008 the P/Book was 2.52 so the stock-weight was:

\[ \text{Stock Weight} = \text{Limit}(1.5 - 0.5 \times \text{P/Book}) = \text{Limit}(1.5 - 0.5 \times 2.52) = \text{Limit}(0.24) \approx 0.24 \]
Example: Adaptive Rebalancing

- On January 12, 2008 the P/Book was 2.52 so stock weight was 0.24.
- From January 2008 to 2009 the S&P 500 lost about (38%).
- US Gov. Bonds yielded about 2.8% in that year.
- Return on the rebalanced portfolio from January 2008 to 2009 was:

\[
\text{Stock Weight} \times \text{Stock Return} + (1 - \text{Stock Weight}) \times \text{Bond Return}
\]
\[
= 0.24 \times (38\%) + (1 - 0.24) \times 2.8\% \approx (7\%)
\]
Medium Risk Rebal. – Annualized Return (1978-2013)

Back-test Medium Risk adaptive rebalancing for all possible starting dates and investment periods up to 10 years during 1978-2013.

Box-plot shows statistics for the annualized return.

This can also be shown in a table...
Medium Risk Rebal. – Annualized Return (1978-2013)

<table>
<thead>
<tr>
<th>Years of Investing</th>
<th>Min</th>
<th>1st Qrt</th>
<th>Median</th>
<th>Mean</th>
<th>3rd Qrt</th>
<th>Max</th>
<th>Stdev</th>
<th>Probability of Loss</th>
<th>Probability &lt; Bond-Only</th>
<th>Probability &lt; Stock-Only</th>
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<tbody>
<tr>
<td>1</td>
<td>(13.9%)</td>
<td>3.9%</td>
<td>6.0%</td>
<td>9.3%</td>
<td>12.8%</td>
<td>65.6%</td>
<td>9.7%</td>
<td>0.06</td>
<td>0.18</td>
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<td>2</td>
<td>(4.3%)</td>
<td>4.8%</td>
<td>6.7%</td>
<td>9.0%</td>
<td>11.9%</td>
<td>33.0%</td>
<td>6.2%</td>
<td>0.02</td>
<td>0.09</td>
<td>0.68</td>
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<tr>
<td>3</td>
<td>(1.1%)</td>
<td>5.0%</td>
<td>7.5%</td>
<td>8.9%</td>
<td>12.2%</td>
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<td>5.4%</td>
<td>0.002</td>
<td>0.07</td>
<td>0.66</td>
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<tr>
<td>4</td>
<td>(0.004%)</td>
<td>5.0%</td>
<td>7.2%</td>
<td>8.8%</td>
<td>12.1%</td>
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<td>5.1%</td>
<td>0.0001</td>
<td>0.05</td>
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<td>4.9%</td>
<td>6.8%</td>
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<td>4.1%</td>
<td>0</td>
<td>0.03</td>
<td>0.81</td>
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Example: Investing for 2 years had mean annualized return 9.0%, min (4.3%), max 33.0%, stdev 6.2%. Investing for 10 years had mean 8.3%, min 2.7%, max 18.1%.
In this 35 year period the Medium Risk adaptive rebalancing performed better than US Gov. Bonds but worse than S&P 500.

But this is not always the case ...
Example: Rebalancing is BETTER Than S&P 500


Investment period is 10 years. Starting date is January 12, 1999.
Example: Rebalancing is WORSE Than S&P 500


Investment period is 10 years. Starting date is August 23, 1990.
Probability of Under-Performance

<table>
<thead>
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<th>(…)</th>
<th>Probability of Loss</th>
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<th>Probability &lt; Stock-Only</th>
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- These are historical probabilities (frequencies) for 1978-2013.
- Probability of loss decreases with longer investment duration.
- Probability of under-performing S&P 500 is high at 0.60-0.84.
Conclusion

• Adaptive rebalancing has several advantages over fixed rebalancing for similar levels of mean annualized return:

• Adaptive rebalancing had much lower probability and magnitude of loss.

• ... and significantly lower probability of under-performing S&P 500 and US Gov. Bonds.

The book also studies other adaptive strategies.