Share Buyback Valuation

Uncertainty

(Part 4)

by

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Stochastic Variables

- The value of a share buyback depends on future earnings.
- Future earnings are uncertain to some degree.
- Uncertainty is modelled mathematically with stochastic variables.
Value WITHOUT Share Buyback

... is the potential for dividend payouts.

Deterministic:

\[ v = Excess \text{ Cash} + \sum_{t=1}^{\infty} \frac{Earnings_t}{(1 + d)^t} \]

\[ V = \frac{v \cdot (1 - TaxDividend)}{Shares} \]

Stochastic variable for \( v \) is denoted \( \mathbb{V} \).
Value WITH Share Buyback

Deterministic:

\[ W = \frac{(v - Buyback) \cdot (1 - TaxDividend)}{Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)} \]

Stochastic:

\[ \mathcal{W} = \frac{V - Buyback}{1 - \frac{Buyback}{MarketCap}} \]
Mean Values (Expected Values)

Mean value WITHOUT share buyback:

\[
E[V] = \sum_v v \cdot Pr[v]
\]

\[
E[V] = \int_0^\infty v \cdot f(v) \, dv
\]

Mean value WITH share buyback:

\[
E[W] = \frac{E[V] - \text{Buyback}}{1 - \frac{\text{Buyback}}{\text{MarketCap}}}
\]
Mean Equilibrium

... is where the mean value to eternal shareholders is unaffected by a share buyback:

\[ E[\mathcal{W}] = E[\mathcal{V}] \iff MarketCap = E[\mathcal{V}] \]

It is usually written as an inequality:

\[ E[\mathcal{W}] > E[\mathcal{V}] \iff MarketCap < E[\mathcal{V}] \]
Relative Value of Share Buyback

Deterministic:

\[
\frac{W}{V} = 1 - \frac{\text{Buyback}}{v} \frac{1}{1 - \frac{\text{Buyback}}{\text{MarketCap}}}
\]

Stochastic:

\[
\frac{W}{V} = 1 - \frac{\text{Buyback}}{V} \frac{1}{1 - \frac{\text{Buyback}}{\text{MarketCap}}}
\]
Mean Relative Value

... is the relative value averaged over all possible outcomes of $V$:

$$E \left[ \frac{W}{V} \right] = \int_0^\infty \frac{1 - \frac{\text{Buyback}}{v}}{1 - \frac{\text{Buyback}}{\text{MarketCap}}} \cdot f(v) \, dv$$

$$= 1 - \text{Buyback} \cdot E \left[ \frac{1}{V} \right]$$

$$= \frac{1 - \text{Buyback}}{1 - \frac{\text{Buyback}}{\text{MarketCap}}}$$
Relative Equilibrium

... is where the mean relative value of a share buyback equals one:

$$E \left( \frac{\mathbb{W}}{\mathbb{V}} \right) = 1 \iff MarketCap = \frac{1}{E \left[ \frac{1}{\mathbb{V}} \right]}$$

It is usually written as an inequality:

$$E \left( \frac{\mathbb{W}}{\mathbb{V}} \right) > 1 \iff MarketCap < \frac{1}{E \left[ \frac{1}{\mathbb{V}} \right]}$$
Minimum Value

If the value to eternal shareholders must increase from a share buyback then the market-cap must be below the minimum possible value for \( \forall \):

\[
\text{MarketCap} < \text{Min}(\forall) \Rightarrow \mathbb{W} > \forall
\]

This is not an equilibrium because equality and bi-implication do not hold.
Equilibrium Relationships

When \( Var[\mathbb{V}] > 0 \) then we know from Jensen’s Inequality:

\[
\frac{1}{E\left[\frac{1}{\mathbb{V}}\right]} < E[\mathbb{V}]
\]

And the harmonic mean is greater than the minimum value:

\[
\text{Min}(\mathbb{V}) < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]}
\]

So the equilibriums are ordered:

Minimum Value < Relative Equilibrium < Mean Equilibrium
Equilibrium Relationships

Mean and relative equilibriums cannot both be satisfied simultaneously.

If $MarketCap = E[V]$ then $E[W] = E[V]$

... but then $E\left[\frac{W}{V}\right] < 1$

This is because of non-linearity of the relative value so potential losses are greater than gains.
Increased Variance

- Variance measures the spread of possible values.
- A share buyback increases the variance of the value to eternal shareholders.

\[ V \text{ar}[W] = \frac{V \text{ar}[V]}{ \left( 1 - \frac{\text{Buyback}}{\text{MarketCap}} \right)^2 } > V \text{ar}[V] \]
Example: Acme Corporation

Assume $\mathbb{V}$ can take on two values with probabilities:

\[ \Pr[\mathbb{V} = $10] = 0.9 \]

\[ \Pr[\mathbb{V} = $1000] = 0.1 \]

Mean value without a share buyback:

\[ E[\mathbb{V}] = \sum_{v} v \cdot \Pr[v] = $10 \cdot 0.9 + $1000 \cdot 0.1 \]

\[ = $109 \]
Mean Equilibrium (Acme Corp.)

... is where the mean value with and without a share buyback are equal:

\[ E[W] > E[V] \iff MarketCap < E[V] = 109 \]
Relative Value (Acme Corp.)

Assume: \( MarketCap = E[V] = $109, Buyback = $5 \)

If \( V = $10 \) (which occurs with probability 0.9):

\[
\frac{W}{V} = \frac{1 - \frac{Buyback}{V}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{5}{10}}{1 - \frac{5}{109}} \approx 52\%
\]

If \( V = $1000 \) (which occurs with probability 0.1):

\[
\frac{W}{V} = \frac{1 - \frac{5}{1000}}{1 - \frac{5}{109}} \approx 104\%
\]
Mean Relative Value (Acme Corp.)

First calculate:

\[
E \left[ \frac{1}{V} \right] = \sum_v \frac{1}{v} \cdot \Pr[V = v] = \frac{1}{10} \cdot 0.9 + \frac{1}{1000} \cdot 0.1
\]

\[
= \frac{901}{10000}
\]

Mean relative value:

\[
E \left[ \frac{W}{V} \right] = \frac{1 - \text{Buyback} \cdot E \left[ \frac{1}{V} \right]}{1 - \frac{\text{Buyback}}{\text{MarketCap}}} = \frac{1 - \frac{5}{109} \cdot \frac{901}{10000}}{1 - \frac{5}{109}} \approx 58\%
\]
Relative Equilibrium (Acme Corp.)

... ensures the mean relative value is greater than one:

\[
E \left[ \frac{W}{V} \right] > 1 \iff \text{MarketCap} < \frac{1}{E \left[ \frac{1}{V} \right]} = \frac{10000}{901} \approx 11.10
\]

Assume: MarketCap = $10.50, Buyback = $5

\[
E \left[ \frac{W}{V} \right] = \frac{1 - \text{Buyback} \cdot E \left[ \frac{1}{V} \right]}{1 - \frac{\text{Buyback}}{\text{MarketCap}}} = \frac{1 - $5 \cdot \frac{901}{10000}}{1 - \frac{$5}{$10.50}} \approx 105\%
\]
Relative Equilibrium is Insufficient (Acme Corp.)

If $V = $10 (which occurs with probability 0.9):

$$\frac{W}{V} = 1 - \frac{Buyback}{V} = 1 - \frac{5}{10} \approx 95\%$$

If $V = $1000 (which occurs with probability 0.1):

$$\frac{W}{V} = 1 - \frac{5}{1000} \approx 190\%$$
Ensure Value Increase (Acme Corp.)

Assume: $\text{MarketCap} = \$9.50$, $\text{Buyback} = \$5$

If $V = \$10$ (which occurs with probability 0.9):

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{V}}{1 - \frac{Buyback}{\text{MarketCap}}} = \frac{1 - \frac{\$5}{\$10}}{1 - \frac{\$5}{\$9.50}} \approx 106\%$$

If $V = \$1000$ (which occurs with probability 0.1):

$$\frac{W}{V} = \frac{1 - \frac{\$5}{\$1000}}{1 - \frac{\$5}{\$9.50}} \approx 210\%$$
Implications

- If a stock’s price equals its expected value to eternal shareholders, then a share buyback would still increase the uncertainty of that value, and any potential losses from the share buyback would be relatively greater than any potential gains.

- So the Dividend Substitution hypothesis, Modigliani-Miller dividend irrelevance hypothesis, and Efficient Market hypothesis are all incorrect.
Summary

• Mean and relative equilibriums are for average outcomes.

• Both equilibriums cannot be satisfied simultaneously.

• Only share buybacks at a market-cap below the minimum possible value ensure that shareholder value is increased.

• A share buyback increases the variance (degree of uncertainty) of the value to eternal shareholders.
Further Reading

This lecture is based on two papers:

**Introduction to Share Buyback Valuation**

**The Value of Share Buybacks**

Both authored by Magnus Erik Hvass Pedersen.

Available on the internet: