# Share Buyback Valuation Uncertainty (Part 4)

by

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# Stochastic Variables

- The value of a share buyback depends on future earnings.
- Future earnings are uncertain to some degree.
- Uncertainty is modelled mathematically with stochastic variables.

# Value WITHOUT Share Buyback

... is the potential for dividend payouts.

Deterministic:

$$v = Excess\ Cash + \sum_{t=1}^{\infty} \frac{Earnings_t}{(1+d)^t}$$

$$V = \frac{v \cdot (1 - TaxDividend)}{Shares}$$

Stochastic variable for $v$ is denoted $\mathbb{V}$.

# Value WITH Share Buyback

Deterministic:

$$W = \frac{(v - Buyback) \cdot (1 - TaxDividend)}{Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)}$$

Stochastic:

$$\mathbb{W} = \frac{\mathbb{V} - Buyback}{1 - \frac{Buyback}{MarketCap}}$$

# Mean Values (Expected Values)

Mean value WITHOUT share buyback:

$$E[\mathbb{V}] = \sum_{v} v \cdot \Pr[v]$$

$$E[\mathbb{V}] = \int_0^\infty v \cdot f(v)\, dv$$

Mean value WITH share buyback:

$$E[\mathbb{W}] = \frac{E[\mathbb{V}] - Buyback}{1 - \frac{Buyback}{MarketCap}}$$

# Mean Equilibrium

... is where the mean value to eternal shareholders is unaffected by a share buyback:

$$E[\mathbb{W}] = E[\mathbb{V}] \Leftrightarrow MarketCap = E[\mathbb{V}]$$

It is usually written as an inequality:

$$E[\mathbb{W}] > E[\mathbb{V}] \Leftrightarrow MarketCap < E[\mathbb{V}]$$

# Relative Value of Share Buyback

Deterministic:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}}$$

Stochastic:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{\mathbb{V}}}{1 - \frac{Buyback}{MarketCap}}$$

# Mean Relative Value

... is the relative value averaged over all possible outcomes of $\mathbb{V}$:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = \int_0^\infty \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} \cdot f(v)\,dv$$

$$= \frac{1 - Buyback \cdot E\left[\frac{1}{\mathbb{V}}\right]}{1 - \frac{Buyback}{MarketCap}}$$

# Relative Equilibrium

... is where the mean relative value of a share buyback equals one:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = 1 \Leftrightarrow MarketCap = \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]}$$

It is usually written as an inequality:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow MarketCap < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]}$$

# Minimum Value

If the value to eternal shareholders must increase from a share buyback then the market-cap must be below the minimum possible value for $\mathbb{V}$:

$$MarketCap < Min(\mathbb{V}) \;\Rightarrow\; \mathbb{W} \;>\; \mathbb{V}$$

This is not an equilibrium because equality and bi-implication do not hold.

# Equilibrium Relationships

When $Var[\mathbb{V}] > 0$ then we know from Jensen's Inequality:

$$\frac{1}{E\left[\frac{1}{\mathbb{V}}\right]} < E[\mathbb{V}]$$

And the harmonic mean is greater than the minimum value:

$$Min(\mathbb{V}) < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]}$$

So the equilibriums are ordered:

Minimum Value < Relative Equilibrium < Mean Equilibrium

# Equilibrium Relationships

Mean and relative equilibriums cannot both be satisfied simultaneously.

If $MarketCap = E[\mathbb{V}]$ then $E[\mathbb{W}] = E[\mathbb{V}]$

... but then $E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] < 1$

This is because of non-linearity of the relative value so potential losses are greater than gains.

# Increased Variance

- Variance measures the spread of possible values.
- A share buyback increases the variance of the value to eternal shareholders.

$$Var[\mathbb{W}] = \frac{Var[\mathbb{V}]}{\left(1 - \frac{Buyback}{MarketCap}\right)^2} > Var[\mathbb{V}]$$

# Example: Acme Corporation

Assume $\mathbb{V}$ can take on two values with probabilities:

$$\Pr[\mathbb{V} = \$10] = 0.9$$

$$\Pr[\mathbb{V} = \$1000] = 0.1$$

Mean value without a share buyback:

$$E[\mathbb{V}] = \sum_{v} v \cdot \Pr[v] = \$10 \cdot 0.9 + \$1000 \cdot 0.1$$

$$= \$109$$

# Mean Equilibrium (Acme Corp.)

... is where the mean value with and without a share buyback are equal:

$$E[\mathbb{W}] > E[\mathbb{V}] \Leftrightarrow MarketCap < E[\mathbb{V}] = \$109$$

# <u>Relative Value (Acme Corp.)</u>

<u>Assume:</u> $MarketCap = E[\mathbb{V}] = \$109$, $Buyback = \$5$

If $\mathbb{V} = \$10$ (which occurs with probability 0.9):

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{\mathbb{V}}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{\$5}{\$10}}{1 - \frac{\$5}{\$109}} \simeq 52\%$$

If $\mathbb{V} = \$1000$ (which occurs with probability 0.1):

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{\$5}{\$1000}}{1 - \frac{\$5}{\$109}} \simeq 104\%$$

# Mean Relative Value (Acme Corp.)

First calculate:

$$E\left[\frac{1}{\mathbb{V}}\right] = \sum_{v} \frac{1}{v} \cdot \Pr[\mathbb{V} = v] = \frac{1}{\$10} \cdot 0.9 + \frac{1}{\$1000} \cdot 0.1$$

$$= \frac{901}{\$10000}$$

Mean relative value:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = \frac{1 - Buyback \cdot E\left[\frac{1}{\mathbb{V}}\right]}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \$5 \cdot \frac{901}{\$10000}}{1 - \frac{\$5}{\$109}} \simeq 58\%$$

# Relative Equilibrium (Acme Corp.)

... ensures the mean relative value is greater than one:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow MarketCap < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]} = \frac{\$10000}{901} \simeq \$11.10$$

Assume: $MarketCap = \$10.50, Buyback = \$5$

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = \frac{1 - Buyback \cdot E\left[\frac{1}{\mathbb{V}}\right]}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \$5 \cdot \frac{901}{\$10000}}{1 - \frac{\$5}{\$10.50}} \simeq 105\%$$

# Relative Equilibrium is Insufficent (Acme Corp.)

If $\mathbb{V} = \$10$ (which occurs with probability 0.9):

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{\mathbb{V}}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{\$5}{\$10}}{1 - \frac{\$5}{\$10.50}} \simeq 95\%$$

If $\mathbb{V} = \$1000$ (which occurs with probability 0.1):

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{\$5}{\$1000}}{1 - \frac{\$5}{\$10.50}} \simeq 190\%$$

# Ensure Value Increase (Acme Corp.)

Assume: $MarketCap = \$9.50,\ Buyback = \$5$

If $\mathbb{V} = \$10$ (which occurs with probability 0.9):

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{\mathbb{V}}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{\$5}{\$10}}{1 - \frac{\$5}{\$9.50}} \simeq 106\%$$

If $\mathbb{V} = \$1000$ (which occurs with probability 0.1):

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{\$5}{\$1000}}{1 - \frac{\$5}{\$9.50}} \simeq 210\%$$

# Implications

- If a stock's price equals its expected value to eternal shareholders, then a share buyback would still increase the uncertainty of that value, and any potential losses from the share buyback would be relatively greater than any potential gains.
- So the Dividend Substitution hypothesis, Modigliani-Miller dividend irrelevance hypothesis, and Efficient Market hypothesis are all incorrect.

# Summary

- Mean and relative equilibriums are for average outcomes.
- Both equilibriums cannot be satisfied simultaneously.
- Only share buybacks at a market-cap below the minimum possible value ensure that shareholder value is increased.
- A share buyback increases the variance (degree of uncertainty) of the value to eternal shareholders.

# Further Reading

This lecture is based on two papers:

**Introduction to Share Buyback Valuation**

**The Value of Share Buybacks**

Both authored by Magnus Erik Hvass Pedersen.

Available on the internet:

**www.Hvass-Labs.Org**