Share Buyback Valuation
Mathematical Analysis of Relative Value
(Part 7)

by

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Why is Mathematical Analysis Important?

• The relative value of a share buyback can be plotted in a graph.
• This shows informally that the relative value is non-linear.
• But graphical plots can be deceiving and we cannot form valid scientific conclusions from just looking at a graphical plot.
• Mathematical analysis uses tools such as algebra and calculus to formally reason about the properties of mathematical functions.
• If the analysis is carried out correctly, then the results can be trusted as scientifically valid.
Value WITHOUT Share Buyback

... is the potential for dividend payouts; that is, the excess cash plus present value of future earnings available for dividend payouts:

\[ v = Excess\ Cash + \sum_{t=1}^{\infty} \frac{Earnings_t}{(1 + d)^t} \]

\[ V = \frac{v \cdot (1 - TaxDividend)}{Shares} \]
Value WITH Share Buyback

A share buyback reduces the cash available for dividends.

... and reduces the number of shares.

\[
W = \frac{(v - \text{Buyback}) \cdot (1 - \text{TaxDividend})}{\text{Shares} \cdot \left(1 - \frac{\text{Buyback}}{\text{MarketCap}} \right)}
\]
Relative Value of Share Buyback

... is the value of a share buyback relative to a dividend payout:

\[
\frac{W}{V} = 1 - \frac{Buyback}{v} \cdot \frac{1}{1 - \frac{Buyback}{MarketCap}}
\]
Limitations of Graphical Plot

Seems to be:

- Non-linear in $Buyback$
- Non-linear in $v$

But:

- Can we be sure of this?
- Are there any surprises?
- What if $v$ is very large?
- How exactly does $W/V$ behave?
Varying the Intrinsic Value

How does the relative value change when we vary the intrinsic value $v$ while holding $MarketCap$ and $Buyback$ constant?

Assume:

- Intrinsic value is positive: $v > 0$
- Buyback amount is between zero and market-cap:

$$MarketCap \geq Buyback \geq 0$$
As $\nu$ approaches positive zero, the limit of $W/V$ is negative infinity:

\[
\lim_{\nu \to 0^+} \frac{W}{V} = \lim_{\nu \to 0^+} \frac{1 - \frac{\text{Buyback}}{\nu}}{1 - \frac{\text{Buyback}}{\text{MarketCap}}} = -\infty
\]
Upper Limit of Relative Value, Varying $\nu$

As $\nu$ approaches positive infinity, the relative value of a share buyback converges to:

$$\lim_{\nu \to \infty} \frac{W}{V} = \lim_{\nu \to \infty} \frac{1 - \frac{\text{Buyback}}{\nu}}{1 - \frac{\text{Buyback}}{\text{MarketCap}}}$$

$$= \frac{\text{MarketCap}}{\text{MarketCap} - \text{Buyback}}$$
Derivative Function, Varying $\nu$

The derivative of $W/V$ with regards to the variable $\nu$ is denoted $\partial_\nu W/V$ which is the rate of change of $W/V$ around a given $\nu$ when the other variables MarketCap and Buyback are held constant:

$$\partial_\nu \frac{W}{V} = \partial_\nu \frac{1 - \frac{\text{Buyback}}{\nu}}{1 - \frac{\text{Buyback}}{\text{MarketCap}}} = \frac{\text{MarketCap} \cdot \text{Buyback}}{\text{MarketCap} - \text{Buyback}} \cdot \nu^{-2}$$

This is continuous and monotonically decreasing for $\nu > 0$. 
Derivative Analysis, Varying \( v \), Upper Limit

As \( v \) approaches positive zero, the limit of \( \partial_v \frac{W}{V} \) is infinity:

\[
\lim_{v \to 0^+} \partial_v \frac{W}{V} = \lim_{v \to 0^+} \frac{\text{MarketCap} \cdot \text{Buyback}}{\text{MarketCap} - \text{Buyback}} \cdot v^{-2} = +\infty
\]

So near \( v = 0 \) the relative value of a share buyback \( \frac{W}{V} \) changes greatly when there is a small change in the value \( v \) and the other variables \( \text{MarketCap} \) and \( \text{Buyback} \) remain constant.
Derivative Analysis, Varying $\nu$, Lower Limit

As $\nu$ approaches infinity, the limit of $\partial_{\nu} \frac{W}{V}$ is zero:

$$
\lim_{\nu \to \infty} \partial_{\nu} \frac{W}{V} = \lim_{\nu \to \infty} \frac{\text{MarketCap} \cdot \text{Buyback}}{\text{MarketCap} - \text{Buyback}} \cdot \nu^{-2} = 0^+
$$

As $\nu$ increases, the relative value of a share buyback $\frac{W}{V}$ changes less.
Derivative Analysis, Varying $\nu$, Graphical Plot

Derivative is continuous and monotonically decreasing. The limits are:

$$\lim_{\nu \to 0^+} \partial_\nu \frac{W}{V} = +\infty$$

$$\lim_{\nu \to \infty} \partial_\nu \frac{W}{V} = 0^+$$
Varying the Buyback Amount

Overpriced Shares
\[ v = 0.7 \cdot MarketCap \]

Underpriced Shares
\[ v = 1.3 \cdot MarketCap \]
If \( Buyback = 0 \) then \( W/V = 1 \).

As \( Buyback \) approaches \( MarketCap \neq v \), the limit of \( W/V \) is either positive or negative infinity:

\[
\lim_{Buyback\to\text{MarketCap}} \frac{W}{V} = \lim_{Buyback\to\text{MarketCap}} \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \pm\infty
\]

If \( MarketCap > v \) then limit is \(-\infty\), if \( MarketCap < v \) then \(+\infty\).
Derivative Function, Varying Buyback

The derivative of $W/V$ for the Buyback variable is the rate of change of $W/V$ around a given Buyback when MarketCap and $v$ are held constant:

$$
\frac{\partial_{\text{Buyback}} W}{V} = \frac{1 - \frac{\text{Buyback}}{v}}{1 - \frac{\text{Buyback}}{\text{MarketCap}}} = \frac{1 - \frac{\text{MarketCap}}{v}}{\frac{\text{Buyback}^2}{\text{MarketCap}} - 2 \cdot \text{Buyback} + \text{MarketCap}}
$$

This is continuous and monotonic when Buyback goes from zero to MarketCap.

If $\text{Buyback} = 0$ then it is $(v - \text{MarketCap})/(v \cdot \text{MarketCap})$.

If $\text{MarketCap} > v$ (overpriced shares) then it is decreasing and has limit $-\infty$.

If $\text{MarketCap} < v$ (underpriced shares) then it is increasing and has limit $+\infty$. 

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Conclusion

• The relative value of a share buyback is a non-linear function with regard to both the intrinsic value $v$ and the buyback amount.
• Buyback of overpriced shares is much more destructive to long-term shareholder value than gains from buyback of underpriced shares.
• This effect is greatly magnified as the buyback amount increases.
Further Reading

This lecture is taken from the paper:

- **The Value of Share Buybacks**

Authored by Magnus Erik Hvass Pedersen.

Available on the internet: