Monte Carlo Simulation
of
A Simple Equity Growth Model

by

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What is Monte Carlo Simulation?

- A computer program simulating thousands of outcomes of a mathematical model.
- This estimates the probability distribution of outcomes.
- Useful when the model cannot be studied analytically.
What is an Equity Growth Model?

• A company retains earnings for investing in new assets.
• The retained earnings are accumulated as equity capital.
• Assume future earnings are related to the equity capital.
• A simple model resamples the historical Return on Equity (ROE) and the historical fraction of earnings retained, and uses this to Monte Carlo simulate the future earnings and equity.
**Equity & Retained Earnings**

Starting equity is normalized to one: \( Equity_0 = 1 \)

Equity at the end of year \( t \) is the previous equity plus retained earnings:

\[ Equity_t = Equity_{t-1} + Earnings_t \cdot (Retain/Earnings)_t \]

Retained earnings are those not paid out as dividends or used for share buyback net of issuance:

\[ \left( \frac{Retain}{Earnings} \right)_t = 1 - \left( \frac{Dividends}{Earnings} \right)_t - \left( \frac{Net Share Buyback}{Earnings} \right)_t \]
Earnings

Earnings for year $t$ are found by multiplying the year’s starting equity by the Return on Equity (ROE):

$$Earnings_t = Equity_{t-1} \cdot ROE_t$$
Price

The price (or market-cap) for time $t$ is calculated from the simulated equity at that time, multiplied by a sample of the historical P/Book (aka Price/Equity):

$$Price_t = (P/Book) \cdot Equity_t$$
Share Buyback and Issuance

Starting number of shares is normalized to one: $Shares_0 = 1$

The number of shares after a share buyback and issuance is:

$$ Shares_t = Shares_{t-1} \cdot \left(1 - \frac{Net \ Share \ Buyback_t}{Price_t} \right) $$
Per Share

The per-share numbers are:

\[\text{Equity Per Share}_t = \frac{\text{Equity}_t}{\text{Shares}_t}\]
\[\text{Earnings Per Share}_t = \frac{\text{Earnings}_t}{\text{Shares}_t}\]
\[\text{Dividend Per Share}_t = \frac{\text{Dividend}_t}{\text{Shares}_t}\]
\[\text{Price Per Share}_t = \frac{\text{Price}_t}{\text{Shares}_t}\]
Value Yield

Assume the shares are held for $n$ years and then sold. The Value Yield is the discount rate that makes the present value of future dividends and present value of the selling share-price equal to the current share-price:

$$Price \ Per \ Share = \sum_{t=1}^{n} \frac{Dividend \ Per \ Share_{t}}{(1 + Value \ Yield)^{t}} + \frac{Price \ Per \ Share_{n}}{(1 + Value \ Yield)^{n}}$$

The value yield is the annualized rate of return on an investment over its holding period, given the current share-price.
Historical Financial Data

All we need for the Monte Carlo simulation of this simple equity growth model is the historical financial data for ROE, Dividends/Earnings, and Net Share Buyback/Earnings. For the pricing model we also need the historical P/Book.
## Wal-Mart, Financial Data

<table>
<thead>
<tr>
<th>Year</th>
<th>Equity (Millions)</th>
<th>Earnings (Millions)</th>
<th>Dividends (Millions)</th>
<th>Net Share Buyback (Millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994</td>
<td>10,753</td>
<td>2,333</td>
<td>299</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>2010</td>
<td>70,468</td>
<td>14,335</td>
<td>4,217</td>
<td>7,276</td>
</tr>
<tr>
<td>2011</td>
<td>68,542</td>
<td>16,389</td>
<td>4,437</td>
<td>14,776</td>
</tr>
<tr>
<td>2012</td>
<td>71,315</td>
<td>15,699</td>
<td>5,048</td>
<td>6,298</td>
</tr>
<tr>
<td>2013</td>
<td>76,343</td>
<td>16,999</td>
<td>5,361</td>
<td>7,600</td>
</tr>
<tr>
<td>2014</td>
<td>76,255</td>
<td>15,918</td>
<td>6,139</td>
<td>6,683</td>
</tr>
</tbody>
</table>

Data from financial reports (SEC Form 10-K).
### Wal-Mart, Financial Ratios

<table>
<thead>
<tr>
<th>Year</th>
<th>ROE</th>
<th>Dividends/ Earnings</th>
<th>Net Buyback/ Earnings</th>
<th>Retain/ Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>25%</td>
<td>15%</td>
<td>0%</td>
<td>85%</td>
</tr>
<tr>
<td>2010</td>
<td>22%</td>
<td>29%</td>
<td>51%</td>
<td>20%</td>
</tr>
<tr>
<td>2011</td>
<td>23%</td>
<td>27%</td>
<td>90%</td>
<td>(17%)</td>
</tr>
<tr>
<td>2012</td>
<td>23%</td>
<td>32%</td>
<td>40%</td>
<td>28%</td>
</tr>
<tr>
<td>2013</td>
<td>25%</td>
<td>32%</td>
<td>45%</td>
<td>24%</td>
</tr>
<tr>
<td>2014</td>
<td>22%</td>
<td>39%</td>
<td>42%</td>
<td>19%</td>
</tr>
</tbody>
</table>

The ratios used in simulation. Calculated from the raw financial data.
Wal-Mart, P/Book

Wal-Mart (1994-2014)

Statistics for 1994-2014:

Mean: 4.7
Stdev: 2.0
Min: 2.4
Max: 12.2
Wal-Mart, Simulated Equity

Equity vs. Year for Wal-Mart equity simulations.
Wal-Mart, Simulated Earnings

![Graph showing Wal-Mart earnings over years]

- X-axis: Year
- Y-axis: Earnings

The graphs display simulated earnings for Wal-Mart over a period of years, with earnings increasing significantly over time.
Wal-Mart, Simulated Dividends
Wal-Mart, Simulated Number of Shares

![Graph showing simulated number of shares over years for Wal-Mart.](image-url)
Wal-Mart, Simulated Price Per Share

Remember this is normalized and must be multiplied by starting equity per share.
Wal-Mart, Value Yield

Value Yield (30 Year Holding)

Value Yield

Holding Period / Years

0.12 0.14 0.16 0.18 0.20

0.0 0.2 0.4 0.6 0.8 1.0 1.2

0 5 10 15 20 25 30

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Warning

There are several limitations of the equity growth model, including:

- The model is simple and may not be suitable for a given company.
- Growth decline should also be modelled or the company may grow bigger than all the combined companies of the S&P 500 index.
- Financial data for more years may be needed.
- Older financial data should perhaps be sampled less frequently.
- The pricing model is crude.

So the simulation results should be interpreted with caution!
Conclusion

- The equity growth model uses historical financial data to simulate future equity, earnings, dividends, etc.
- The simulated equity is used with samples of the historical P/Book to estimate future stock-prices.
- This is a new paradigm for Monte Carlo simulation in finance.
- The model has several limitations and should be used with caution.
- The model can be extended – please do so and share your results!
Further Reading

This talk is based on the papers:

- Monte Carlo Simulation in Financial Valuation
- Portfolio Optimization and Monte Carlo Simulation

Authored by Magnus Erik Hvass Pedersen.

Available on the internet:

www.Hvass-Labs.Org